INTRODUCTION

- Significant Figures
- Significant Figures in Calculations
- Types of Errors
- Principal Definitions and Equations to Report in the Lab Report

Significant Figures are all the digits that are known accurately plus the one estimated digit.

Rules for Identifying Significant Figures

1. Nonzero digits are always significant.

Length = 1.351 cm – (4 sig. fig)

2. Final or ending zeros written to the right of the decimal point are significant.

Length = 1.30 cm (3 sig. fig.)

3. Zeros written between significant figures are significant.

Length = 1.03 cm (3 sig. fig.)

4. Zeros written to the right of the decimal point for the purpose of spacing the decimal point are not significant.

Mass = 0.005 kg = 5 * 10^{-3} kg (1 sig. fig.)

Significant Figures in Calculations

Adding/Subtracting

When numbers are added (subtracted), the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum (difference).

Example:

- ✓ **1.002 – 0.998 = 0.004**
- ✓ **123.2 + 5.35 = 128.5**
- ✓ **123 + 5.35 = 128**

Multiplying/Dividing

In multiplying (dividing) two or more quantities, the number of significant figures in the final product is the same as in the factor that is having the lowest number of significant figures.

Example:

- ✓ **12.71 m * 3.46 m = 43.976 m^2 → 44.0 m^2**
- ✓ **12.71 m / 3.4 m = 3.738 → 3.7**

Types of Errors

Random or Statistical Error

This class of error is produced by unpredictable or unknown variations in the measuring process.

The effect of the random error may be reduced by repetition of the experiment.

Systematic Error

This class of error is commonly caused by a flaw in the experimental apparatus.

For example, a bad calibration in the instrumentation will give a systematic error.

Personal Error or Mistakes

This class of error can be completely eliminated if the experimenter is careful enough.

Principal Definitions and Equations

"**Mean**" – of the *N* measurements, is the best approximation to the true value.

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i = \frac{1}{N}(x_1 + x_2 + \ldots x_n)$$

"Deviation" – tells how much the i^{th} measurement x_i *differs from the average* $\bar{x}$.

$$x_i - \bar{x} = d_i$$

The average of the deviations $\bar{d}$ will be zero for any set of measurements because d_i is sometimes positive and sometimes negative in just such a way that $\bar{d}$ is zero.

"Standard deviation" of the measurements σ_x – is an estimate of the average uncertainty of the measurements**.**

$$\sigma_x = \sqrt{\frac{1}{(N-1)}\sum_{i=1}^{N}(d_i)^2} = \sqrt{\frac{1}{N-1}\sum(x_i - \bar{x})^2}$$

"Standard deviation of the mean" gives the uncertainty in the final answer. It is denoted $\sigma_{\bar{x}}$ or Δx.

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$$

Reporting of the Results

The standard form for reporting a measurement of a physical quantity *x* is:

$$x = \bar{x} \pm \sigma_{\bar{x}}$$

Best estimate for *x*

Uncertainty of the measurements

OR

$$x = \bar{x} \pm \left(\frac{\sigma}{\bar{x}} \times 100\% \right).$$

Percentage/relative error

This statement expresses our confidence that the correct value of *x* probably lies

in (or close to) the range from $x = \bar{x} - \sigma_{\bar{x}}$ to $x = \bar{x} + \sigma_{\bar{x}}$.

Rules for Reporting the Experimental Results

- **Experimental uncertainties should be rounded to one sig.fig.**
- **The last sig.fig. in the result should be in the same decimal place as the uncertainty.**

Example: $l = 24.245 \pm 0.006$ mm

Error propagation **(An Introduction to Error Analysis John R. Taylor)**

Sums and Differences

If several quantities *x,…, w* are measured with uncertainties *δx,…, δw* and the measured values are used to compute

$$q = x + \dots + z - (u + \dots + w),$$

then the uncertainty in the computed value of *q* is the sum,

$$\Delta q = \Delta x + \dots + \Delta z + \Delta u + \dots + \Delta w,$$

of all the original uncertainties.

Uncertainty in Products and Quotients

If several quantities *x,…, w* are measured with uncertainties *δx,…, δw* and the measured values are used to compute

$$q = \frac{x \times \ldots \times z}{u \times \ldots \times w},$$

then the fractional uncertainty in the computed value of *q* is the sum,

$$\frac{\Delta q}{|q|} = \frac{\Delta x}{|x|} + \ldots + \frac{\Delta z}{|z|} + \frac{\Delta u}{|u|} + \ldots + \frac{\Delta w}{|w|},$$

of the fractional uncertainties in *x,…, w.*

Uncertainty for a Measured Quantity Times Exact Number

If the quantity *x* is measured with uncertainty δ*x* and is used to compute the product

$$q = Bx,$$

where *B* has no uncertainty, then the uncertainty in *q* is just:

$$\Delta q = |B|\,\Delta x.$$

Uncertainty in Power

If the quantity *x* is measured with uncertainties *δx* and the measured values are used to compute the power

$$q = x^n,$$

then the fractional uncertainty in *q* is *n* times that in *x*,

$$\frac{\Delta q}{|q|} = n\frac{\Delta x}{|x|}.$$

PERCENT OF DISCREPANCY - D

$$D = \frac{|accepted\ value - experimental\ value|}{accepted\ value} \times 100\%$$